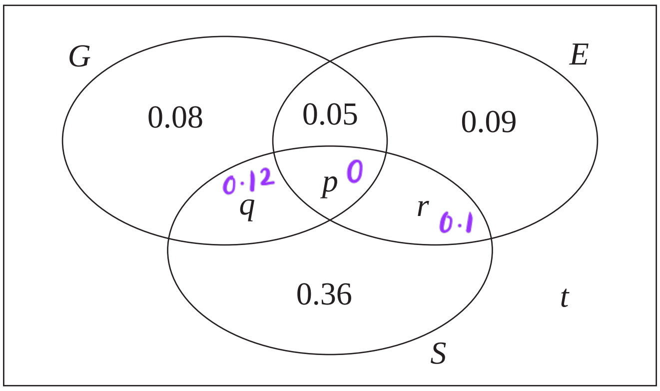
1. A large college produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the college is selected at random and the events G, E and S are defined as follows

G is the event that the student reads the magazine about green issues
E is the event that the student reads the magazine about equality
S is the event that the student reads the magazine about sports

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines.
(1)

No students read all three magazines and $P(G) = 0.25$

(b) Find

(i) the value of p

(ii) the value of q
(3)

Given that $P(S \mid E) = \frac{5}{12}$

(c) find

(i) the value of r

(ii) the value of t
(4)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.
Show your working clearly.
(3)

a) P(reads exactly one magazine) = 0.08 + 0.09 + 0.36

= 0.53 1

b) $P(G \cap E \cap S) = 0$, $P(G) = 0.25$

(i) $P(G \cap E \cap S) = p$

$= 0$ $\therefore p = 0$ (1)

(ii) $P(G) = 0.08 + q + p + 0.05 = 0.25$ (1)

$0.13 + q = 0.25$

$\therefore q = 0.12$ (1)

(c) $P(S|E) = \frac{5}{12}$ (1)

(i) $\frac{r + p}{r + p + 0.09 + 0.05} = \frac{5}{12}$ (1)

$\frac{r}{r + 0.14} = \frac{5}{12}$

$12r = 5r + 0.70$

$7r = 0.70$

$r = 0.10$ (1)

(ii) Σ probabilities $= 1$: $0.08 + 0.05 + 0.09 + q + p + r + 0.36 + t = 1$

$0.08 + 0.05 + 0.09 + 0.12 + 0 + 0.1 + 0.36 + t = 1$

$0.80 + t = 1$

$\therefore t = 0.20$ (1)

d) for independent events, $P(A \cap B) = P(A)P(B)$

$P(S \cap E') = 0.36 + q$

$= 0.36 + 0.12$

$= 0.48$ (1)

$P(G) = 0.25$

$P([S \cap E'] \cap G) = q$

$= 0.12$ (1)

$0.12 = 0.48 \times 0.25$

$0.12 = 0.12$

(1)

$\therefore (S \cap E')$ and G are independent events

2. A company has 1825 employees.
The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, A or B, where the employees live

	A	***B***
Professional	740	380
Skilled	275	90
Elementary	260	80

An employee is chosen at random.

Find the probability that this employee

(a) is skilled,
(1)

(b) lives in area B and is not a professional.
(1)

Some classifications of employees are more likely to work from home.

1 • 65% of professional employees in both area A and area B work from home
2 • 40% of skilled employees in both area A and area B work from home
3 • 5% of elementary employees in both area A and area B work from home

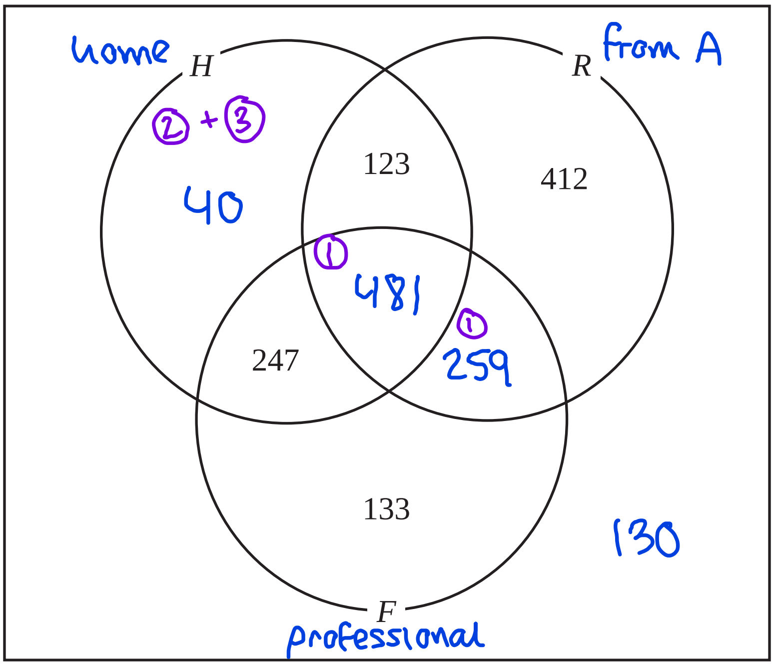
- Event F is that the employee is a professional
- Event H is that the employee works from home
- Event R is that the employee is from area A

(c) Using this information, complete the Venn diagram on the opposite page.
(4)

(d) Find $P(R' \cap F)$
(1)

(e) Find $P([H \cup R]')$
(1)

(f) Find $P(F \mid H)$
(2)



①
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Turn over for a spare diagram if you need to redraw your Venn diagram.

a) $P(\text{skilled}) = \frac{275+90}{1825} = \frac{1}{5}$ ①

b) $P(\text{B and not professional}) = \frac{90+80}{1825} = \frac{34}{365}$ ①

c) ① 740 professional from A, 65% work from home
$740 \times 0.65 = 481$ ①

② 90 skilled from B, 40% work from home
$90 \times 0.4 = 36$

③ 80 elementary from B, 5% work from home
$80 \times 0.05 = 4$

$36 + 4 = 40$ non professionals from B who work from home

① 740 professional from A, 35% do not work from home: $740 \times 0.35 = 259$

$1825 - (40 + 123 + 412 + 481 + 247 + 259 + 133)$

$= 130$

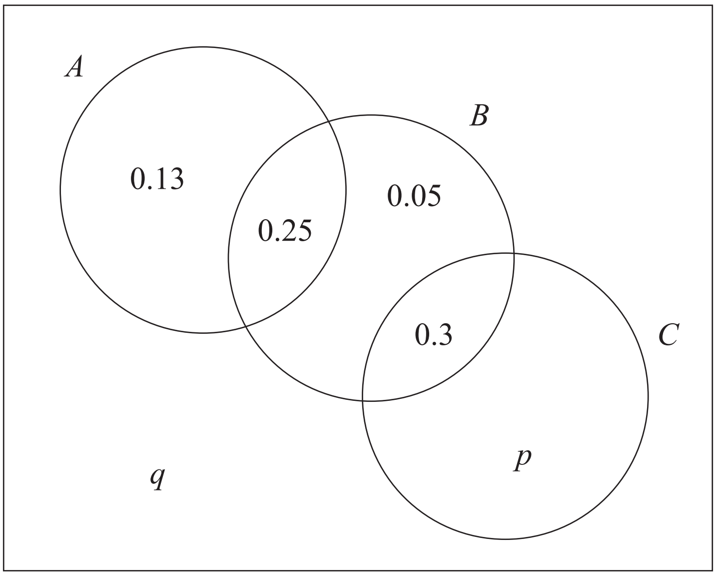
d) $P(R' \cap F) = \dfrac{247 + 133}{1825} = 0.208 \ (3sf)$ ①

e) $P([H \cup R]') = \dfrac{133 + 130}{1825} = 0.144 \ (3sf)$ ①

f) $P(F \mid H) = \dfrac{P(F \cap H)}{P(H)} = \dfrac{247 + 481}{40 + 123 + 247 + 481}$ ①

$= 0.817 \ (3sf)$ ①

3. The Venn diagram, where p and q are probabilities, shows the three events A, B and C and their associated probabilities.



(a) Find P(A) **(1)**

The events B and C are independent.

(b) Find the value of p and the value of q **(3)**

(c) Find P($A|B'$) $\frac{P(A \cap B')}{P(B')}$ **(2)**

a) $P(A) = 0.13 + 0.25$

$= 0.38$ (1)

b) $P(B \cap C) = P(B) \times P(C)$ — independent event

$0.3 = (0.3 + 0.25 + 0.05) \times (0.3 \times p)$ (1)

$0.3 = 0.6 \times (0.3 + p)$

$0.3 = 0.18 + 0.6p$

$0.12 = 0.6p$

$\therefore p = 0.2$ (1)

Σ probabilities $= 1$:

$0.13 + 0.25 + 0.05 + 0.3 + p + q = 1$

$0.13 + 0.25 + 0.05 + 0.3 + 0.2 + q = 1$

$0.93 + q = 1$

$\therefore q = 0.07$ (1)

(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')}$

$= \dfrac{0.13}{0.13 + 0.2 + 0.07}$ (1)

$= \dfrac{0.13}{0.4}$

$= 0.325$ (1)

4. Tisam is playing a game.
She uses a ball, a cup and a spinner.

The random variable X represents the number the spinner lands on when it is spun.
The probability distribution of X is given in the following table

x	20	50	80	100
$P(X = x)$	a	b	c	d

where a, b, c and d are probabilities.

To play the game

- the spinner is spun to obtain a value of x
- Tisam then stands x cm from the cup and tries to throw the ball into the cup

The event S represents the event that Tisam successfully throws the ball into the cup.

To model this game Tisam assumes that

- $P(S \mid \{X = x\}) = \dfrac{k}{x}$ where k is a constant
- $P(S \cap \{X = x\})$ should be the same whatever value of x is obtained from the spinner

Using Tisam's model,

(a) show that $c = \dfrac{8}{5}b$

(2)

(b) find the probability distribution of X

(5)

Nav tries, a large number of times, to throw the ball into the cup from a distance of 100 cm.
He successfully gets the ball in the cup 30% of the time.

(c) State, giving a reason, why Tisam's model of this game is not suitable to describe Nav playing the game for all values of X

(1)

a) To find equation with c and b, we use data when $x = 50$ and $x = 80$.

$P(S \cap \{X = 50\}) = P(S \cap \{X = 80\}) = \text{Constant}$

$P(S \mid \{X = x\}) = P(S)$

$P(S \cap \{X = x\}) = P(S) \times P(X = x)$

$= P(S \mid \{X = x\}) \times P(X = x)$

$= \frac{k}{x} \times P(X = x)$

when $x = 50$ and $x = 80$,

$\frac{k}{50} \times b = \frac{k}{80} \times c$ (1)

$c = \frac{80}{50} b$

$\therefore \quad c = \frac{8}{5} b$ (1) (shown)

find the probabilities in term of a.

b) $b = \frac{5}{2} a$ (1) , $c = 4a$, $d = 5a$ (1)

Σ probabilities $= 1$:

$a + b + c + d = 1$

$2 \times \left(a + \frac{5}{2} a + 4a + 5a \right) = 1 \times 2$ (1)

$\therefore \quad 2a + 5a + 8a + 10a = 2$

$\therefore \quad 25a = 2$

$\therefore \quad a = \frac{2}{25}$ (1)

$\therefore b = \frac{\cancel{5}^1}{\cancel{2}_1}\left(\frac{\cancel{2}^1}{\cancel{25}_5}\right) = \frac{1}{5}$

$\therefore c = 4\left(\frac{2}{25}\right) = \frac{8}{25}$

$\therefore d = \cancel{5}^1\left(\frac{2}{\cancel{25}_5}\right) = \frac{2}{5}$ (1)

x	20	50	80	100
$P(X=x)$	$\frac{2}{25}$	$\frac{1}{5}$	$\frac{8}{25}$	$\frac{2}{5}$

c) $P(S \mid \{X=20\}) = \frac{k}{20}$

$\therefore \frac{k}{100} = 0.3$

$\therefore k = 30 \Rightarrow \frac{30}{20} = 1.5$ (which is > 1)

(1)

$\therefore$ For a distance of 20 cm, this would give a probability of greater than 1, which is impossible.